Speed-of-Light Exit Velocity Propulsion Method

Eric Holmes[1] and Nathanael Snow[2]
Auburn University, Auburn, AL

These documents serve as validation and guidelines for designing the working end of a gamma ray thruster. The performance of the device is determined by current quantum electrodynamics and classical engine performance theories. This conceptual modeling approach is used to determine the behavior of positronium as it is being produced, separated, and undertaking the annihilation process. The most important parameters being the exit velocity of ejected mass, and the consequential thrust output. This analysis can be used to map the behavior of the deflected gamma rays as a function of incoming electron velocity and the initial photon wavelength. This theory was initially analyzed using MATLAB and Python, but will be modeled in greater depth using more detailed algorithms. New theorized systems, such as the consumable mass introducer will be evaluated. The performance will be compared with engines already in use, and using this data, it will be determined if such a rocket will be efficient and useful for in-space propulsion. This document is intended to survey the possibility of speed-of-light exit velocities and a new means of electron-positron production, in amounts appreciable to generate thrust, are of particular interest.

Nomenclature

c = speed of light in vacuum
E_e = incoming electron energy
$E_{e'}$ = outgoing electron energy
E_γ = incoming photon energy
$E_{\gamma'}$ = outgoing photon energy
h = Plank's constant
m_e = electron rest mass
n_e = atomic-electron number density
n_{hot} = hot electron number density
n_i = atomic number
n_γ = photon number density
r_o = classical electron radius
Z = atomic number
α = fine structure constant (~1/137)
α *(angle)* = angle between incident photon and electron in the nozzle
β = auxiliary angle (difference between initial electron and outgoing photon directions in nozzle)
γ = incoming electron Lorentz factor
θ = photon scattering angle (can be in pair annihilation and nozzle)
λ = incoming photon wavelength
λ' = outgoing photon wavelength
v_e = electron velocity
ϕ = electron recoil angle in nozzle
$\phi_1(y)$, $\phi_2(y)$= screening factor as function of screening parameter γ

[1] Undergraduate Researcher, Aerospace Engineering Department, Auburn AL, Student Member
[2] Undergraduate Researcher, Aerospace Engineering Department, Auburn AL, Student Member

I. Introduction

The future of life on Earth, more specifically human life, can either remain bound by the limitations of Earth or it can evolve into a multi-planetary species. As per statistical probability, a catastrophic event will eventually occur on Earth and extinguish any trace of all organisms. Logic would deduce that at some point we must locate and travel to a habitable celestial body; however, observed and unobserved Earth-like planets exist light years outside the solar system. Current propulsion methods are limited to only a short range from Earth, thus we must explore alternative methods of propulsion for long term missions. This paper is to serve as an introduction to show that gamma ray propulsion should be considered as a plausible solution to this problem. Although this transportation method may not be a viable form of human transport in its early stages it should still be investigated as a propulsion method for robotic vehicles and potentially crewed vessels in the distant future.

The active systems necessary to operate and maintain the device are the powerhouse, magnetic field generator, laser, relativistic electron beam lasers and consumable material introducer and storage units. The powerhouse will be used to power each component of the device and will incorporate a way to recycle excess power to increase the overall efficiency of the device. A petawatt class laser will be aimed incident to the high Z material to produce positronium, and the material will need to be replaced as it is being degraded. A material introducer will feed new material in sync with the pulse of the laser. Magnetic fields will be introduced to the process to split the positronium pairs and redirect them according to the wanted geometry of the pair annihilation process. In conjunction with the electric field from the laser, the magnetic field will also be used to increase the rate of pair production by directing the B-field perpendicular to the laser's E-field. The electron beam will function as a nozzle by reflecting the gamma rays from the annihilation process to produce thrust.

II. Particle Interactions

There are three main hot spots of particle interactions in the system: the pair production, annihilation and deflection process. Each of these processes in theory, can be geometrically optimized for energy densities and scattering angles. The angular and energy differential cross sections can be analyzed for each process, which, if done using the methods below, can determine which angles have the highest probability of particles scattering into it, given incoming photon and electron energy data.

A. Nozzle

A gamma ray is inherently difficult to control due to its nature to pass uninhibited through most mediums, and in the meantime disturbing the atoms it passes through. A traditional nozzle will not work for proper gamma ray reflection as no known material exhibits characteristics of such an interaction. The theory behind modeling the acceleration of the propellant of this engine at its core involves detailed relativistic electron Compton Scattering analysis, which is a plausible means of gamma ray reflection. Traditional Compton Scattering involves a high-energy photon incident upon an electron assumed to be at rest energy, and this process has been explained to a very high degree of accuracy. Arthur Holly Compton derived formulas detailing the geometry of the interaction, the Klein-Nishina formula, predict the scattering angle probability. In our case, both the photon and electron have high energy, therefore a different differential cross section equation was used to determine the scattering angle probability as per Eq. 1. This equation takes into account that the initial electron velocity is assumed to be moving.

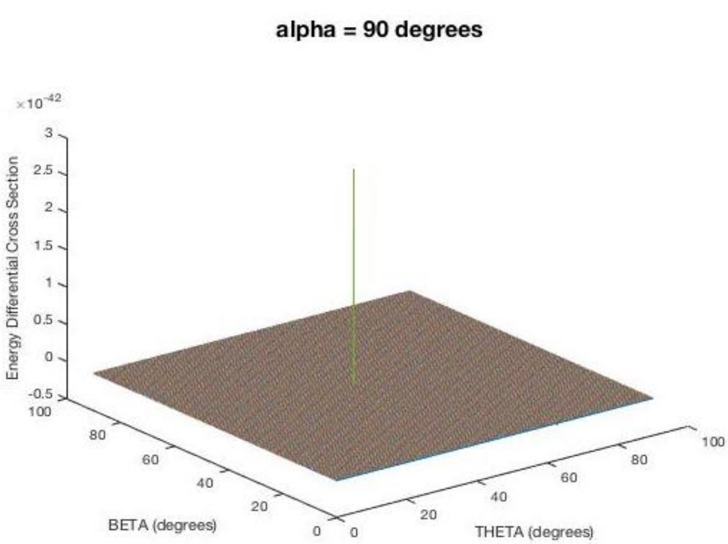

Figure 1: Initial test of the differential cross section at an input angle of α = 90°.

In theory, if the electron has a high enough energy, the gamma ray can be scattered such that it does not lose any energy or possibly even gain energy in the scattering process. If this is the case, this could be useful to high-energy photon propulsion purposes.

To determine the structure and geometry of the "nozzle", one must obtain precise geometry of the gamma ray reflection to obtain accurate values of thrust. An analysis of the differential cross section of relativistic electron Compton Scattering, Eq. 1, must be performed to obtain the highest probability of scattering at a certain angle. It was found through numerical analysis in MATLAB that the highest probability of a predictable scattering angle is where theta = 41.85˚, beta = 48.15˚ and alpha = 90˚, as shown in Figure 1. These values of highest probability were found by calculating the final photon energy ($E_{\gamma'}$), found in Eq. 2, verses every possible alpha, beta, and theta angle, to a thousandths place degree of accuracy. The energy differential cross section equation, Eq. 1, for a relativistic electron colliding with a high-energy photon is shown below. From Fig. 2, when the electron beam is at a 60˚ angle to the photons, the probability is very distributed. This indicates that there could still be analytical uncertainty and a higher degree of accuracy need be made in the calculations to further explore the highest probability.

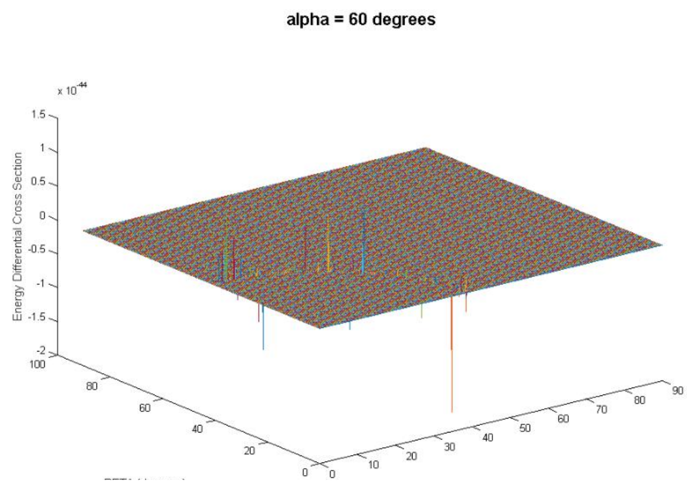

Figure 2: Probability when α = 60

$$\frac{d\sigma}{dE_{\gamma'}} = \frac{\pi r_o^2}{2}\frac{m_e^2}{E_\gamma E_e^2}\left[\left[\frac{m_e^4}{4E_\gamma^2 E_e^2}\times\left(\frac{E_{\gamma'}^2}{E_e - E_{\gamma'}}\right)^2\right] - \left[\frac{m_e^2}{E_\gamma E_e}\times\frac{E_{\gamma'}}{E_e - E_{\gamma'}}\right] + \frac{E_e - E_{\gamma'}}{E_e} + \frac{E_e}{E_e - E_{\gamma'}}\right] \tag{1}$$

$$\lambda' = \frac{\frac{h}{m_e\gamma}(cos\theta - 1) + \lambda(v_e cos\beta - c)}{v_e cos\alpha - c} \tag{2}$$

$$\phi = acos\left[\frac{\frac{h}{\lambda} + \gamma m_e v_e cos\alpha - \frac{h}{\lambda'}cos\theta}{\left[h^2\left(\frac{1}{\lambda^2} + \frac{1}{\lambda'^2} - \frac{2}{\lambda\lambda'}\right) + 2h\gamma m_e c\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) + m_e^2 c^2\left(\gamma^2 - 1\right)\right]^{\frac{1}{2}}}\right] \tag{3}$$

The behavior of the scattering process is a function of both space and time and thus requires the use of a differential cross section to accurately model the interaction. The cross section is used to map the likelihood of the interaction between the gamma ray and a photon. The methodology is as follows: with the highest probability of the scattering angles theta and beta, and input angle alpha, find the probable scattered photon wavelength using Eq 2. Once the scattered photon wavelength is found, the electron recoil angle can be found from Eq. 3, note that this gives two angles so there is a degree of uncertainty to this angle. It was found that the typical electron recoil angle from the values above is ϕ = 42.8828°.

These recoiled electrons are the subject of the possibility of power reusability. A mechanism which can absorb the recoiled electrons for possible reuse of energy will be further explored. For maximum efficiency, the recycling may need to be an active system, but may benefit more from the decreased cost of a passive system. The photon and electron absorbers themselves should be a passive system. Figure 3 shows the angles and their corresponding measurements; these angles are the resultant highest probability angle calculations from the MATLAB and Python codes, thus this depicts what a typical geometry might look like.

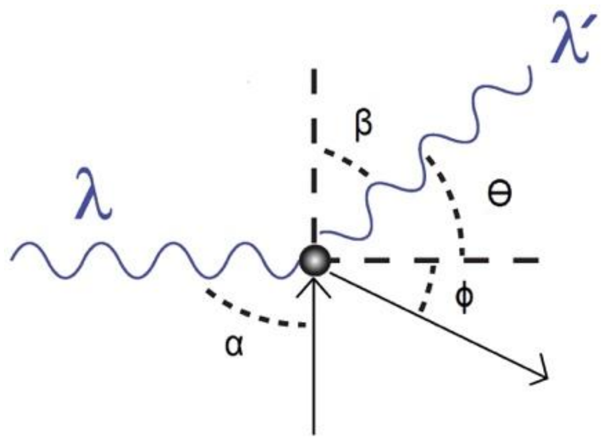

Figure 3: Interaction Geometry

B. Pair Production

The theory behind the antimatter production involves the Trident and Bethe-Heitler processes. Each of these occur when the electric field of a sufficiently intense laser interacts with the Coulomb field of the high-Z target's atoms, which in turn creates positronium and bremsstrahlung. Typically, the laser will have a minimum intensity of 10e+18 W/cm^2 for these particles to form. According to recent research, source rates of up to 10e+24 positrons per seconds can be formed given a petawatt laser system is used and the target is a Platinum class metal[4]. Because no known method for storing large amounts of antimatter are available, the on-board production method should be considered, given the material introducer and petawatt laser systems operate in harmony. It was found that there is a 2-step process for creating positronium at the cost of creating less pairs, thus direct incidence on the target (the 1-step approach) is the better option for maximum pair yield rates[1].

The Trident and Bethe-Heitler processes can be explained using differential cross sections for each process along with the respective pair yield rate. The Trident process involves direct creation of pairs by the energetic electrons interactions with the Coulomb field of the host atom's nucleus. The Bethe-Heitler process describes the indirect creation of pairs by creation of a bremsstrahlung photon first then photon creation in the nuclear field (photo-pair production). The Bethe-Heitler process is the much less prevalent process. In theory, a sufficiently strong magnetic field perpendicular to the E-field of the laser would increase the rate that positronium pairs would be created. It is important to know exactly how these particles are moving and how they are interacting with each other and the environment so accurate calculations of total pair production rates can be found.

C. Pair Annihilation

The pair annihilation process is the source of the energy output for the nozzle. Basic classical electrodynamics says that an electron and positron will annihilate and produce two photons with .511MeV of energy. The positronium pairs from the Trident and Bethe-Heitler processes must be split into their constituent particles, an electron and a positron. A sufficiently strong magnetic field must separate the pairs before they annihilate; ortho-positronium has a half-life of approximately 1.244e-10 seconds[8]. Studies show that this is possible, such as Hyperfine splitting, or the splitting of positronium via a sufficiently energetic photon colliding with the positronium atoms[9]. This benefit may not be worth the required new subsystem integration into the device; the complexity of such a system will be overwhelming due to the large amounts of positronium pairs being created. Conservation of linear momentum says that an electron and a positron, with a net linear momentum of zero and low energy, will annihilate and produce two photons whose directions are directly opposite of one another. High energy annihilation can result in production of other particles, such as D and B mesons[10]. If the annihilation process is low energy, then the amount of power required by the magnetic field generators will be minimized. Studies will investigate the possibility of geometrical changes in scattering due to a change in net linear momentum being greater than zero. The geometrical changes can be explained through quantum electrodynamics theories for solving for the differential cross section for the annihilation process. The cross section is directly dependent on the two particles' kinetic energies and the angular difference between their travel. There are methods for finding angles of highest probability, as was done with the nozzle calculations in MATLAB and Python for the pair annihilation process.

III. Required Systems

The active systems required to operate and maintain control of a gamma ray thruster are extensive, as are the components necessary to operate and maintain the vehicle housing the device. Each component will need to operate uninhibited by any other part of the engine and be resistant to the magnetic fields, gamma ray radiation, etc. The different parts necessary for the device consist of the powerhouse, magnetic field generator, laser, relativistic electron beam lasers, consumable material introducer and storage units.

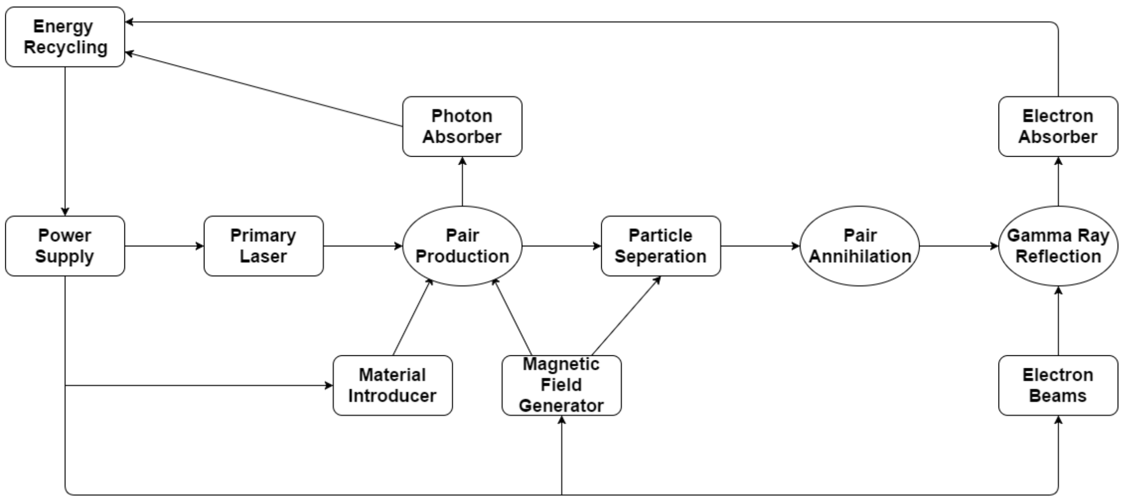

Figure 4: Engine Block Diagram

The powerhouse will be used to power each component of the device and the overall power required for operation is dependent upon the net power to be used by all the systems in the vehicle. The final requirement cannot be conclusively determined without more experimental data, and therefore would require a physical mockup to get an accurate gauge. Ideally, the power supply will have a high enough efficiency to sustain necessary systems concurrently, in addition to the passive recycling components, the electron absorbers in the nozzle and the bremsstrahlung absorbers from the pair production process, to ensure a longer available duration of use. Recycling excess power would also increase the overall efficiency of the device.

The laser will be the most taxing upon the power supply and will need to be on the order of a petawatt class to be most effective. It is aimed incident to the high Z material to produce positronium, and the material will be need to be replaced with the material introducer as it is being spent. Space required to house the laser is about 1500 sq. ft., but will be condensed in its later form to fit in the fuselage of the device housing. With current laser technology, it is not possible to produce pairs in a vacuum (without the high-Z target there), as the lasers are not intense enough. With a high-Z target 'catalyst', it would need a peak power of a PW, or around 10e+18 W/cm^2, but the average power required is very low[1]. It should be possible to produce PW pulses with 10% wall-plug efficiency and a few Hertz. With finely tuned optics, it is possible to reach 2-3x higher efficiency and a much higher frequency using advanced materials and some optics development. There is a significant difference between a 10J/10fs PW or a 1000J/1000fs PW, and 1KJ pulses at 1kHz with 10% efficiency will require a 10MW power source. There is plenty of room for improvement here, both in terms of refining the process through experimentation and implementing new theories and technologies as they are found.

The target that the laser is focused on will be a finite region, thus the effective surface area that the material must expose can be sized accordingly. This material will dissolve over time due to the added energy from the laser, and will need to be replaced when there is not enough usable area left to produce the electron pairs. The timing of when and how much material is replaced will depend upon the frequency and diameter of the laser pulses, as well as the thermal properties of the material. The targets are very small, around 3 mm in diameter[2].

The reservoir for the material will be used to store the entire contents of the high-Z target needed for the mission. It will need to be insulated from the magnetic field, gamma rays, and other products of the engine during its operation to maintain the life of the material and its effectiveness. The proximity of the plenum with respect to the system introducing it will affect the complexity of the delivery system and therefore the weight. The system must not interact with the laser, and studies will go into how this will be accomplished using current QED theories along with other quantum theories and classical theories as needed.

The magnetic field strength will need to be strong enough to overcome the 6.8 eV volt binding energy of positronium. It will need to split the electron and positrons before they annihilate, so an investigation need be made in to the different positronium states; the 2S metastable state has the longest time to exist on average before annihilation. Another means of positronium splitting, hyperfine splitting, supports the claim that the two constituent particles can be split by a sufficiently energetic photon, but the practicality of this is much less attractive compared to the magnetic field. In conjunction with the electric field from the laser, the magnetic field will also be used to increase the rate of pair production. The electron beam will function as a nozzle by reflecting the gamma rays from the annihilation process to produce thrust. Again, laser-magnetic field interaction will be studied.

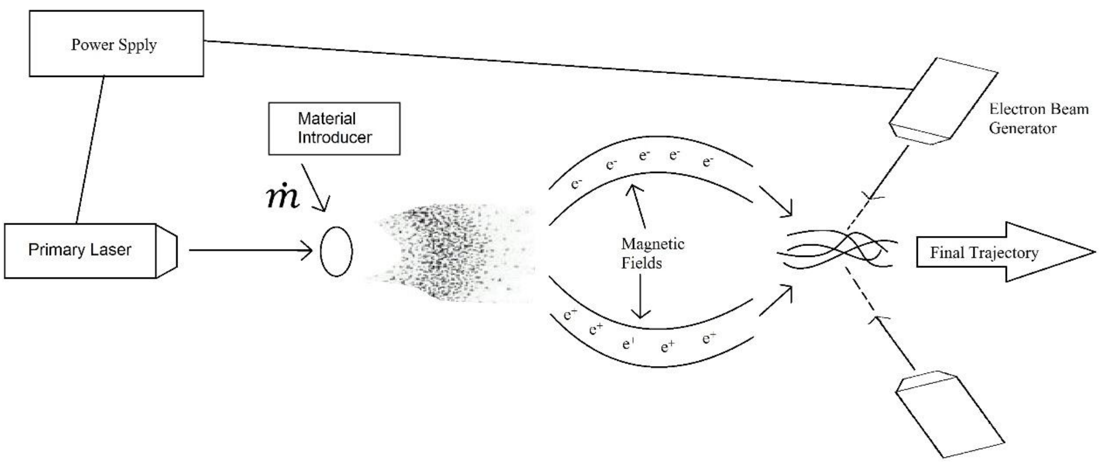

Figure 5: Engine Schematic

The electron beam lasers serve as the “reflection” method for the gamma rays in the nozzle section. Using single particle quantum electrodynamic methods, the angle at which the beam should be incident on the photons from the annihilations process can be optimized and found. The electrons will need to be relativistic; the velocities we are finding in our numerical studies, to sufficiently reflect the gamma rays, must be on the order of ~1e8 m/s. There will be two electron beam lasers, one reflecting each corresponding gamma ray beam from the annihilation process.

IV. Vehicle Performance

To determine if this engine will be efficient and useful, classical performance parameters need to be determined and compared to current propulsion methods. Thrust, specific impulse and delta-V are all important fundamental characteristics for analysis. Using the rocket equation with exit velocities at the speed of light and the particle flow rate, thrust output and specific impulse can be found. An initial analysis was performed using a 200,000kg initial vehicle mass and a laser burn time of a year, the thrust was found to be 37455 N with an I_{sp} of 30569232 s. The percent speed of light achieved by this case was a staggering 1.9897 %. The apparent mass of a gamma ray can be found from the highest probable energy of the scattered gamma ray and solving for the mass component of $E = mc^2$. Assuming conditions where the magnetic field yields the optimal annihilation, the estimated rate is 1.0e+24 positrons per second. This will profit double that quantity in gamma rays per second, and thus the mass flow rate is determined by the product of the particle flow rate and the apparent mass of gamma rays. The amount of propellant

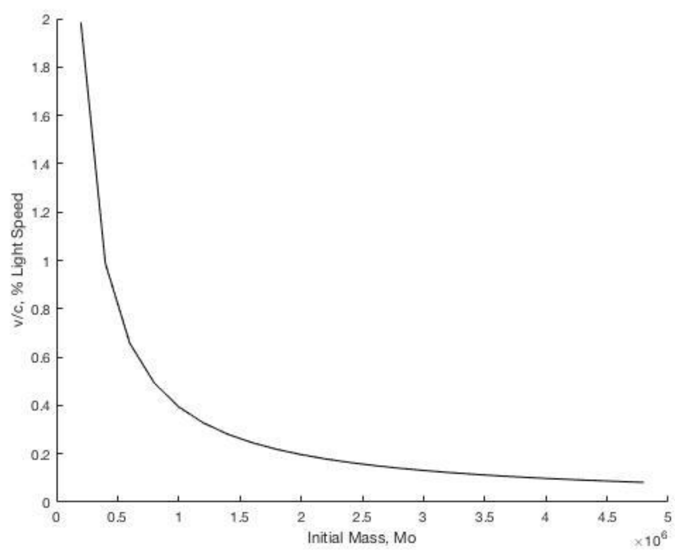

Figure 6: Initial Mass vs Percent of Light Speed

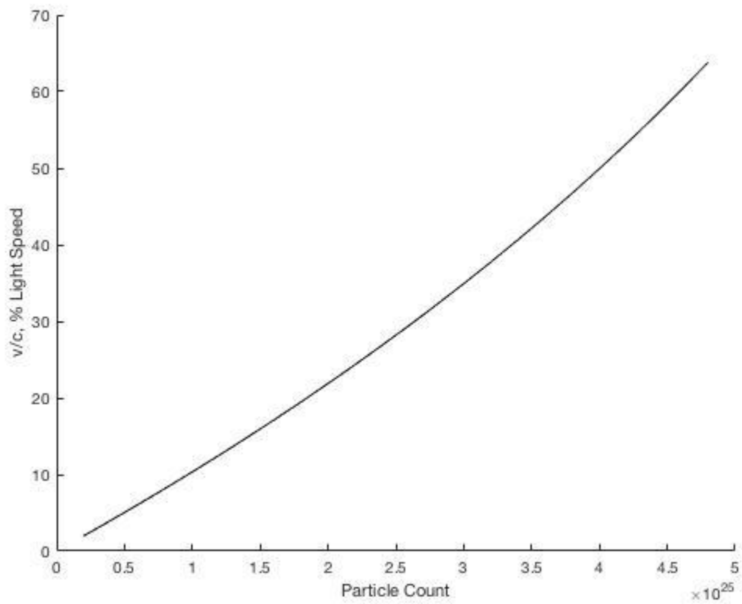

Figure 7: Particle Count vs Percent Light Speed

needed to run the engine for a year at the calculated mass flow rate of 1.2493e-4 kg/s would result in approximately four metric tons. The propellant mass fraction would be on the order of 0.0197. Due to its independence from any chemical change, the device is restricted only by its weight and particle flow rate. Increasing the magnitude of the produced positronium will return an exponential increase in thrust output. Intuitively, an increase in mass would decrease the capable velocity of the spacecraft. If it were to be used for an in-space propulsion system for a vehicle that weighed only a million kilograms, which is five times larger than the proposed initial mass, the positronium output would need to be one hundred times larger. If one were interested in moving a large enough payload to explore another solar system or enough resources to inhabit another planet, it would require massive scaling of the device's components. A scale on that magnitude would require an immense amount of Platinum class material, which would demand a significant fraction of the initial mass be dedicated to the propulsive material. Not only must a high volume of the required disposable material exist, but laser, materials and energy storing technology need to be advanced further to achieve optimal efficiency. Thus, the limiting factor of the capability of this propulsion method would rely on the pair output of the material verses the power required for the laser and storable energy. Figure 9 relates the thrust output to the velocity of the input electron from the annihilation process and it was found that there is a pair of optimal velocities to maximize performance. The first peak indicates that initial electron velocity optimization is worthwhile, and that there is a specific electron velocity lower than the speed of light that can maximize thrust output. It would also be of interest for small scale vehicles, such as satellites using the propulsive device for thrust vectoring or maintaining a certain orbit. The higher energy case would be much more difficult to maintain because the electron velocity would be at the speed of light, and therefore would only be viable if the absolute maximum thrust was necessary. The strength of the free electron lasers may be marginally lower for the first peak, but would need to be of substantial forte to accelerate the electrons to their maximum speed.

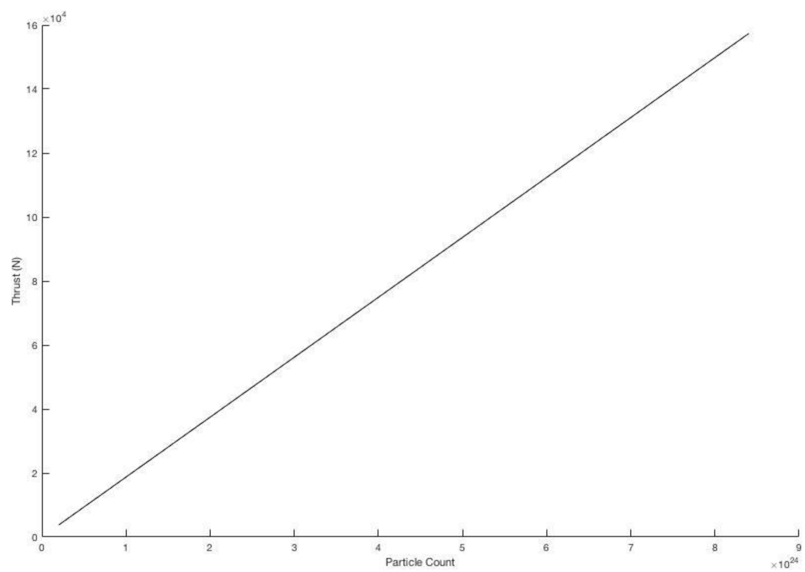

Figure 8: Particle Count vs Thrust

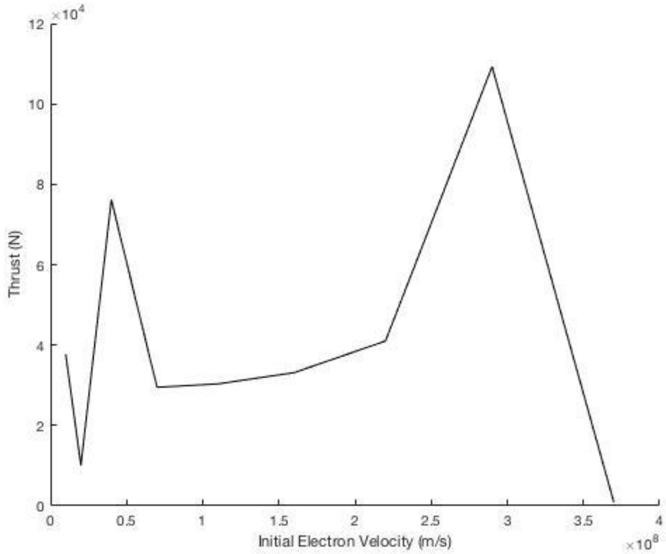

Figure 9: Initial Electron Velocity vs Thrust

Conclusion

When considering methods to accomplish speed of light exit velocities, one must consider a system that deals with photons, whose inherent velocity is the speed of light. Using gamma rays as a form of propellant is a plausible system to have appreciable amounts of thrust. Matter-antimatter annihilation, more specifically electron-positron annihilation is a viable method to produce gamma rays in a sizeable quantity. Currently, no known method of storing large amounts of antimatter is available, so on-board production using the high-intensity laser system has been proposed. Once the initial analysis of the different differential cross sections is completed in MATLAB or another programming language, the numerical system can slowly be put together and optimized. The final power requirement will be found from this optimized information.

A device that benefits from on-board antimatter production in the proposed method would experience exit velocities nearly the speed of light. The use of such a system enables space travel to cover distances that are orders of magnitude further than current propulsion methods can attain. For this device to reach fruition, the limits of human exploration need be pushed further, likewise expanding human knowledge and keeping the natural explorer in all of us alive.

References

Electronic Publications

[1]E. Liang, T. Clarke, A. Henderson, W. Fu, W. Lo, D. Taylor, P. Chaguine, S. Zhou, Y. Hua, X. Cen, X. Wang, J. Kao, H. Hasson, G. Dyer, K. Serratto, N. Riley, M. Donovan, T. Ditmire. "High e+/e- Ratio Dense Pair Creation with 10^21 W.cm^-2 Laser Irradiating Solid Targets." *Scientific Reports* 14 09 2015.

[2]Hui Chen, A. Link, Y. Sentoku, P. Audebert, F. Fiuza, A. Hazi, R. F. Heeter, M. Hill, L. Hobbs, A. J. Kemp, G. E. Kemp, S. Kerr, D. D. Meyerhofer, J. Myatt, S. R. Nagel, J. Park, R. Tommasini, G. J. Williams. "The scaling of electron and positron generation in intense laser-solid interactions." *Physics of Plasmas*. Vol. 22. AIP, 2015. Journal.

[3]Ian Hsu, C. - C. Chu, C. - I. Yu, C. - I. Chen, A. - T. Lai and Y. - C. Liu#, P. - K. Tseng, G. - Y. Hsiung,R. -C. Hsu, C. -P. Wang, R. - C. Chen. "Energy Measurement of Relativistic Electrons by Compton Scattering." scientific. Institute of Nuclear Science, National Tsing-Hua University andSynchrotron Radiation Research Center, 1996.

[4]LLE Review. "Optimizing Electron–Positron Pair Production on kJ-Class High-Intensity Lasers for the Purpose of Pair-Plasma Creation." Vol. 116. n.d. Review.

[5]Ryoichi Hajima, Mamoru Fujiwara. "Narrow-band GeV photons generated from an x-ray free-electron laser oscillator." Review. National Institutes for Quantum and Radiological Science and Technology, 2016.

[6]Sun, Changchun. "Characterizations of a Compton scattering gamma ray beam and diagnostics of an electron beam." Ed. Duke University. 04 09 2009. Lecture.

[7]Toshiki Tajima, William Brocklesby, Gerard Mourou. "The Next Laser Powerhouse." *Optics and Photonics Reviews* May 2013.

[8]Branson, Jim. *Positronium*. 22 04 2013. UCSD. <http://quantummechanics.ucsd.edu/ph130a/130_notes/node365.html>.

[9]—. *Positronium*. 22 04 2013. UCSD. <http://quantummechanics.ucsd.edu/ph130a/130_notes/node365.html>.

Unpublished Papers and Books

[10]Thomson, Mark. "Handout 4: Electron-Positron Annihilation." 2011. PowerPoint.